Medium effects in K$^+$ nucleus interaction from consistent analysis of integral and differential cross sections

E. Friedman[a], A. Gal[a] and J. Mareš[b]

[a] Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

[b] Nuclear Physics Institute, 25068 Řež, Czech Republic

Abstract

Self consistency in the analysis of transmission measurements for K$^+$ on several nuclei in the momentum range of 500-700 MeV/c is achieved with a '$t_{eff}(\rho)\rho$' potential and new results are derived for total cross sections. The imaginary part of the t_{eff} amplitude is found to increase linearly with the *average* nuclear density in excess of a threshold value of 0.088±0.004 fm^{-3}. This phenomenological density dependence of the K$^+$ nucleus optical potential also gives rise to good agreement with recent measurements of differential cross sections for elastic scattering of 715 MeV/c K$^+$ by ^{6}Li and C.

PACS: 25.80.Nv, 13.85.Lg, 13.75.-n, 21.30.Fe

KEYWORDS: K$^+$ nucleus total and reaction cross sections 500-700 MeV/c on ^{6}Li, C, Si, Ca. Self consistent analysis. Medium effects.

E-MAIL: elifried@vms.huji.ac.il, avragal@vms.huji.ac.il, mares@ujf.cas.cz

Total cross sections for the interaction of 500-700 MeV/c K^+ with several nuclei were derived from transmission experiments performed in recent years [1–4]. Since in this energy range the KN interaction is relatively weak and does not vary strongly with energy, one expects that optical potentials close to the '$t\rho$' approximation will be adequate. Consequently such potentials indeed were used for extracting total cross sections from the data [1–4]. However, attempts to reproduce these total cross sections with optical potentials, including in addition various degrees of conventional nuclear physics sophistication beyond the $t\rho$ starting point, invariably gave rise [5] to calculated values smaller than the measured cross sections. This as well as earlier indications led to speculations about nonconventional medium modifications of the KN interaction [6–10]. Very recently the same transmission experiments [4] were re-analyzed [11] to yield reaction cross sections, for the first time, and also revised values for the total cross sections.

The derivation of reaction (σ_R) and total (σ_T) cross sections from transmission measurements involves the use of an optical potential V_{opt}. An obvious question of self consistency is whether these derived σ_R and σ_T values are consistent with values predicted by V_{opt}, or more generally, do optical potentials which are constructed to fit σ_R and σ_T values lead to the same values (within errors) when used in a re-analysis of transmission experiments. The answer, unfortunately, has been negative so far. As an example we quote results for Si at 714 MeV/c; the derived reaction and total cross sections are [11] 317.5$\pm$3.6 and 396.5$\pm$2.3 mb respectively whereas the corresponding calculated values, based on a $t\rho$ potential, are 253 and 328 mb respectively. Attempting to fit the data for ^{6}Li, C, Si and Ca with $t_{eff}\rho$, or with density-dependent potentials that include higher powers of ρ, generally reduces the values of χ^2 per point from a few hundreds to a few tens. However, if any of these modified potentials is used in reanalyses of the transmission measurements then the process *diverges*. It is therefore obvious that the question of self consistency is essential to the derivation of integral cross sections from transmission measurements and to the proper handling of systematic errors. In the present work we report on an empirical specific density dependent potential that achieves excellent fits to the measured reaction and total cross sections within a fully

consistent picture. In addition it fits well recent measurements [12] of angular distributions for the elastic scattering of 715 MeV/c K$^+$ by ^{6}Li and C.

The interaction of K$^+$ with nuclei is described in the present work by the Klein Gordon equation

$$\left[\nabla^2 + k^2 - (2\varepsilon_{red}^{(A)} V - V_c^2)\right] \psi = 0 \quad (\hbar = c = 1) \tag{1}$$

where k and $\varepsilon_{red}^{(A)}$ are the wave number and reduced energy respectively in the $c.m.$ system, $(\varepsilon_{red}^{(A)})^{-1} = E_p^{-1} + E_t^{-1}$ in terms of the $c.m.$ energies for the projectile and target particles, respectively. V_c is the Coulomb potential due to the charge distribution of the nucleus, and $V = V_c + V_{opt}$. The simplest possible $t\rho$ form was chosen for the optical potential, namely

$$2\varepsilon_{red}^{(A)} V_{opt}(r) = -4\pi F_k b_0 \rho(r), \qquad F_k = \frac{M_A \sqrt{s}}{M(E_t + E_p)} \quad , \tag{2}$$

where F_k is a kinematical factor resulting from the transformation of amplitudes between the KN and the K$^+$ nucleus $c.m.$ systems and b_0 is the value of the KN scattering amplitude in the forward direction. M is the free nucleon mass, M_A is the mass of the target nucleus, $\sqrt{s}$ is the total projectile-nucleon energy in their $c.m.$ system and the nuclear density distribution $\rho(r)$ is normalized to A, the number of nucleons in the target nucleus. Another choice of kinematics and wave equation will be mentioned below.

It was shown in [11] that by comparing the ratios of measured to calculated σ_R and σ_T values (based on $t\rho$ potentials) for C, Si and Ca to the corresponding ratios for ^{6}Li, a clear dependence on the nucleus emerges that appears to be independent of beam momentum. An alternative way of displaying the dependence of σ_R and σ_T on the nuclear medium, beyond the obvious dependence on A and ρ implied by eqs.(1) and (2), is to fit the parameter b_0 of V_{opt} to the ^{6}Li data and then use this potential to *calculate* reaction and total cross sections for the other, much denser nuclei. Figure 1 shows ratios of experimental to calculated integral cross sections for C, Si and Ca. These ratios deviate considerably from the value of one in a way which is largely independent of the beam momentum.

The choice of as light a nucleus as ^{6}Li to serve as a basis for studying K meson nuclear medium effects in terms of optical potentials might appear questionable. Nevertheless, the

use of a potential can be bypassed at the relatively high energies encountered here by using Glauber's eikonal multiple scattering expansion. For example, in very light nuclei the bulk contribution to the total cross section comes from single scattering, i.e. $\sigma_T \sim A\sigma$, where σ is the KN total cross section. For the double scattering contribution at 714 MeV/c we calculate for ^{6}Li a value of -5 ± 2 mb compared to the single scattering contribution of 84 mb. The uncertainty of ± 2 mb reflects uncertainty in the value of b_0 (particularly its real part) and modifications due to nuclear center of mass motion and Pauli and shell model correlations which provide the longest range two-body nuclear correlations. The neglect of $1/A$ corrections involved in using an eikonalized optical potential to approximate the multiple scattering expansion does not apply to the dominant single scattering term. Consequently it amounts to only about 1 mb in the present case. Thus we estimate that the use of an optical potential at these energies for forward K^+ ^{6}Li scattering does not introduce errors larger than about 2%. This is considerably smaller than the medium effects in the heavier nuclei discussed in the present work.

Since integral cross sections are sensitive particularly to the imaginary part of the optical potential, we have tried in a first step to see whether a modification of the imaginary part of a $t_{eff}\rho$ optical potential, via multiplication by a rescaling factor, separately for each target nucleus, is capable of providing good fits to the data. Excellent agreement with the C, Si and Ca data was achieved when the imaginary potential was increased by 17-25%, depending on the nucleus, on top of the increase of 5-15% in ImV_{opt}, with respect to the free KN interaction, needed to fit the ^{6}Li data as described above. Moreover, this target-dependent rescaling factor turned out to be independent of the beam momentum. Next we proceeded to replace this *ad hoc* approach by a more general model that depends on parameters of the target nuclei. We failed to reach good agreement with the data by using the local density in modifying the potential. However, it was possible to achieve good fits to the data by correlating ImV_{opt} with the *average* nuclear density, defined as follows:

$$\overline{\rho} = \frac{1}{A} \int \rho^2 d\mathbf{r}. \qquad (3)$$

Values of $\overline{\rho}$ can be obtained quite reliably from Hartree-Fock calculations or from simpler single-particle calculations [13,14] that are constrained by r.m.s. radii of nuclear charge distributions. The results for the nuclei considered here are 0.049, 0.103, 0.110 and 0.107 fm^{-3} for ^{6}Li, C, Si and Ca respectively, with an uncertainty of about $\pm 1\%$ for the last three cases. The enhancement of ImV_{opt} was then assumed to take place above a threshold average density ρ_c, by multiplying the imaginary part of the conventional potential eq.(2) by the rescaling factor $1 + \beta(\overline{\rho} - \rho_c)\Theta(\overline{\rho} - \rho_c)$, and a fit was made at each momentum varying the parameters b_0, β and ρ_c. The last two parameters were found to have the same values at all four beam momenta. Table I shows that excellent fits are obtained within this linear rescaling model. In contrast, attempting to replace the threshold function $\Theta(\overline{\rho} - \rho_c)$ by a power of the average density failed to produce fits to the data. The average values of ρ_c=0.088$\pm$0.004 fm^{-3} and β=13.0$\pm$3.4 fm^3 were used in these calculations. This value of ρ_c is considerably larger than $\overline{\rho}(^6\mathrm{Li})$ so that the ^{6}Li data are fitted, as above, merely by adjusting the parameter b_0. Note that the inequality $\overline{\rho}$(Ca)$>$ $\overline{\rho}$(C) (see above), which implies stronger rescaling of ImV_{opt} for Ca than for C, does not translate into larger σ(exp.)/σ(calc.) values (see fig.1) for Ca than for C since multiple scattering is more effective in the former. The departure of the fitted (medium) KN forward scattering amplitude b_0 from its free-space value (given in parentheses in table I) is worth a comment. Imb_0 is found to gradually increase with energy up to 16% above its free-space value. This increase is quantitatively consistent with the estimates of Garcia-Recio *et al.* [10] of the contribution due to meson exchange currents. However, we have no clues for the theoretical significance of ρ_c in providing a threshold density above which new reaction channels in the K-nucleus interaction open up, beyond those accounted for by the reactive content of the impulse approximation. As for Reb_0, we observe from table I a clear tendency for *less* repulsive KN interaction with increasing energy than that implied by the free space values. Such additional *attraction* could be expected due to the proximity of the K*N and KΔ channels.

The next question is that of self consistency. With the above enhancement of ImV_{opt} we re-analyzed all the transmission measurements and obtained new values for σ_R that almost

agree, within errors, with the previous values based on a $t\rho$ potential [11]. Values of σ_T are higher a little more (3-5%) than the previous ones [11], as expected on grounds of the extent of theoretical input involved in the analysis of transmission measurements [11,15]. Fits to these new σ_R and σ_T values left the parameters of table I essentially unchanged and another round of re-analysis of the transmission measurements showed that full convergence had been achieved, at all four beam momenta. The parameters in table I correspond to the converged, self consistent values of σ_R and σ_T.

A further test of the whole procedure is now possible at the highest momentum thanks to the very recent publication of experimental differential cross sections for K^+ elastic scattering at 715 MeV/c on ^{6}Li and C [12]. These angular distributions were therefore used in optical model fits, with a variety of potentials including the one of table I. The normalization of the data was allowed to vary too, within the quoted range of $\pm$15%. The best fits to the elastic scattering data without rescaling ImV_{opt} achieved reasonable fits to the angular distributions but with χ^2 per point of about 20 for the reaction and total cross sections (for ^{6}Li, C, Si and Ca) at this momentum. In contrast, when ImV_{opt} was rescaled as in table I, values of χ^2 per point for the reaction and total cross sections were in the range of 1.6 to 2.0. The normalization factor for the angular distribution data was well determined at the acceptable value of 0.92$\pm$0.05. Figure 2 shows the good agreement between calculated and measured angular distributions. The potential of fig.2 was then used in a further re-analysis of the transmission measurements at 714 MeV/c to yield yet another set of reaction and total cross sections, based this time on potentials that fit also the available angular distributions. Table II summarizes σ_R and σ_T values as obtained from transmission measurements using (a) the $t\rho$ potential (eqs.(1) and (2)) and (b), (c) two potentials incorporating the $\Theta(\overline{\rho}- \ \rho_c)$ rescaling, where (c) is also constrained by fits to differential cross sections for elastic scattering. For (b) and (c) the process converges, which is not the case for (a). Values of σ_T are larger by 3-5% than those found in non self-consistent analyses. Values of σ_R remain practically unchanged.

One final comment on the kinematics and wave equation is in order. Since there is no

agreed relativistic wave equation to be used with a finite mass target once a potential is introduced, we have checked the dependence of our results on the wave equation and kinematics by repeating all the calculations with the Goldberger-Watson equation [16]. Details do change but the overall picture remains unchanged. In particular, there is a threshold average nuclear density of about 0.09 fm^{-3} above which the imaginary potential must be rescaled in order to achieve agreement between calculated and measured integral cross section within a consistent analysis. A major advantage of using the wave equation (1) with the potential (2) is that it yields in the eikonal approximation an attenuation factor which agrees with the semiclassical expression $(\sigma\rho)^{-1}$ for the mean free path. This ensures the correct reactive content of the procedure adopted here.

To summarize, empirical rescaling of the imaginary part of the optical potential with the *average nuclear density in excess of a threshold value* solves the long standing problem of inconsistencies in the extraction of reaction and total cross sections from transmission measurements for the interaction of 500-700 MeV/c K^+ with nuclei. The self consistent values of total cross sections are 3-5% larger than previously published values. This potential is also very successful in reproducing the available angular distributions of elastically scattered K^+ by ^{6}Li and C. The empirical finding that the absorptive part of V_{opt} increases with the average nuclear density above a threshold value ρ_c, may signal that novel degrees of freedom are excited in K^+ nucleus interactions.

This research was partially supported by the Israel Science Foundation administered by the Israel Academy of Sciences and Humanities (E.F.) and by the U.S.-Israel Binational Science Foundation (A.G.). A.G. would like to thank B.K. Jennings for his hospitality at TRIUMF, and J.M. acknowledges the hospitality of the Racah Institute of Physics at the Hebrew University.

REFERENCES

[1] Y. Mardor *et al.*, Phys. Rev. Lett. 65 (1990) 2110.

[2] R.A. Krauss *et al.*, Phys. Rev. C46 (1992) 655.

[3] R. Sawafta *et al.*, Phys. Lett. B307 (1993) 293.

[4] R. Weiss *et al.*, Phys. Rev. C49 (1994) 2569.

[5] C.M. Chen and D.J. Ernst, Phys. Rev. C45 (1992) 2011; M.F. Jiang, D.J. Ernst and C.M. Chen, Phys. Rev. C51 (1995) 857.

[6] P.B. Siegel, W.B. Kaufmann and W.R. Gibbs, Phys. Rev. C30 (1984) 1256; C31 (1985) 2184.

[7] G.E. Brown, C.B. Dover, P.B. Siegel and W. Weise, Phys. Rev. Lett. 60 (1988) 2723.

[8] J.C. Caillon and J. Labarsouque, Phys. Rev. C45 (1992) 2503 ; Phys. Lett. B295 (1992) 21; Nucl. Phys. A572 (1994) 649 ; Nucl. Phys. A589 (1995) 609; Phys. Rev. C53 (1996) 1993.

[9] M.J. Jiang and D. S. Koltun, Phys. Rev. C46 (1992) 2462.

[10] C. Garcia-Recio, J. Nieves and E. Oset, Phys. Rev. C51 (1995) 237.

[11] E. Friedman *et al.*, Phys. Rev. C, in press.

[12] R. Michael *et al.*, Phys. Lett. B382 (1996) 29.

[13] C.J. Batty and G.W. Greenlees, Nucl. Phys. A133 (1969) 673.

[14] D.J. Millener and P.E. Hodgson, Nucl. Phys. A209 (1973) 59.

[15] M. Arima and K. Masutani, Phys. Rev. C47 (1993) 1325.

[16] M.L. Goldberger and K.M. Watson, Collision Theory, John Wiley (New York, 1964) §6.8.

FIGURES

FIG. 1. Ratios between experimental and calculated cross sections for calculations based on fits to ^{6}Li. Full squares represent σ_R and open circles represent σ_T.

FIG. 2. Fits to angular distributions for elastic scattering of 715 MeV/c K^+ by ^{6}Li and C (see text).

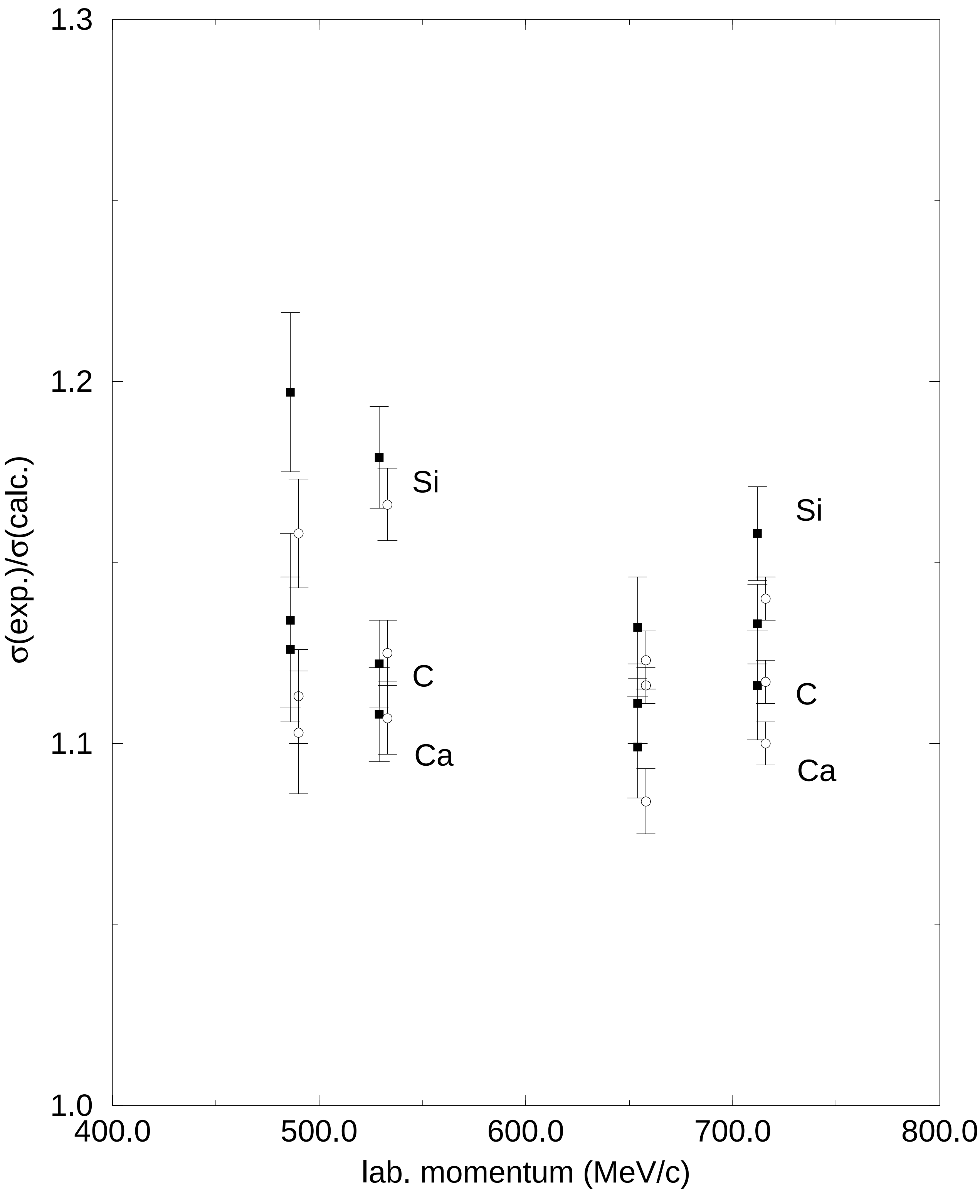
TABLES

TABLE I. Fits to K^+ nucleus σ_R and σ_T values obtained by rescaling ImV_{opt} by the factor $1+\beta(\overline{\rho}-\rho_c)\Theta(\overline{\rho}-\rho_c)$ with ρ_c=0.088 fm^{-3}, β=13.0 fm^3. Values in parentheses are for the free KN interaction.

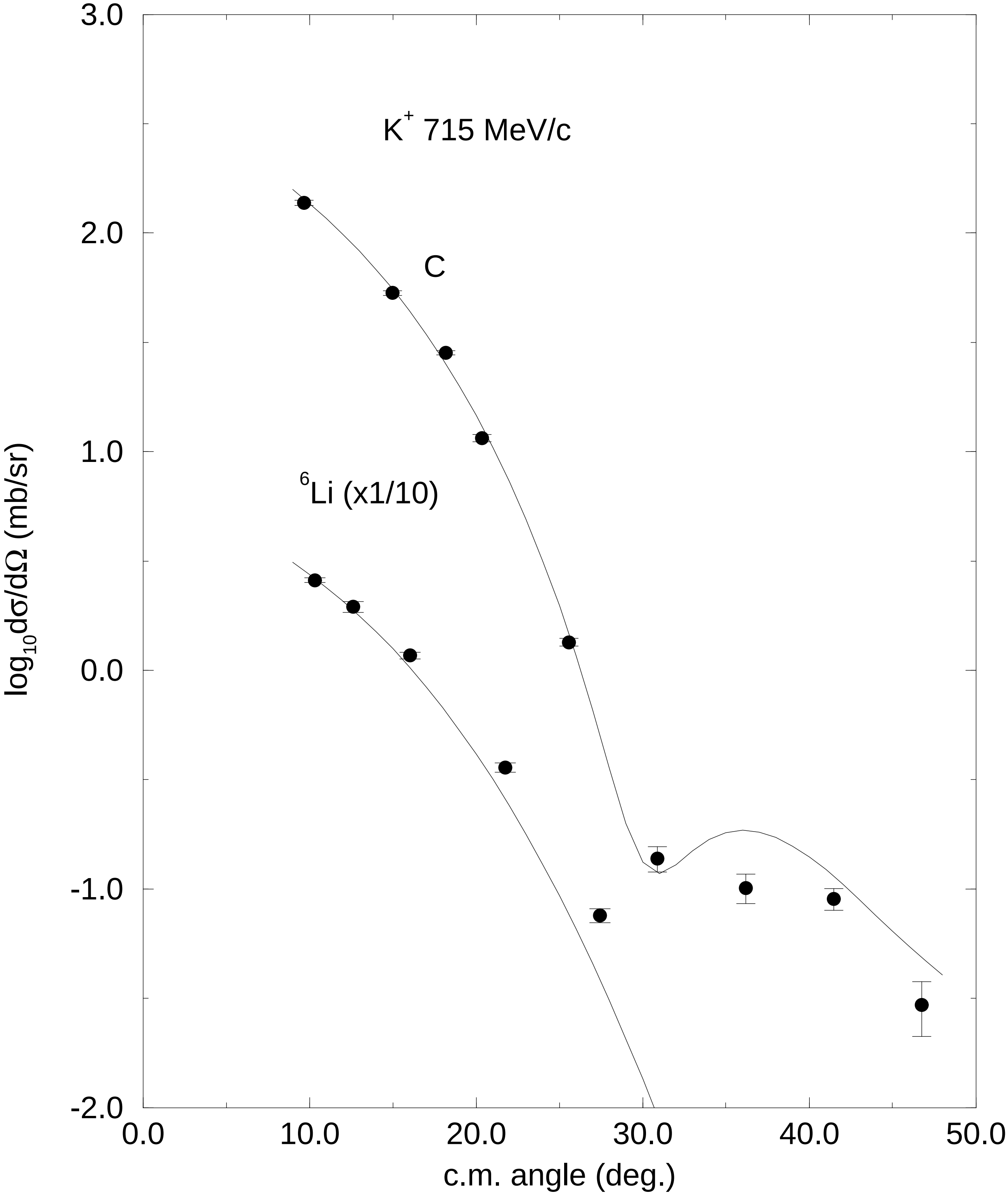
p(MeV/c)	Reb_0(fm)	Imb_0(fm)	χ^2/N
488	-0.154±0.012	0.160±0.002	0.6
	(-0.178)	(0.153)	
531	-0.119±0.012	0.186±0.002	1.2
	(-0.172)	(0.170)	
656	-0.035±0.062	0.241±0.002	0.2
	(-0.165)	(0.213)	
714	-0.044±0.064	0.265±0.001	1.0
	(-0.161)	(0.228)	

TABLE II. K^+ nucleus reaction and total cross sections derived from transmission measurements at 714MeV/c, based on potentials (a), (b) and (c) (see text).

potl.	σ_R (mb)				σ_T (mb)			
	Li	C	Si	Ca	Li	C	Si	Ca
(a)	80.7±1.2	151.8±1.5	318.7±3.6	413.7±5.5	86.8±0.6	177.4±0.9	392.0±2.3	523.2±2.8
(b)	82.2±1.2	152.8±1.5	320.2±3.6	417.1±5.5	88.5±0.6	183.8±0.9	411.3±2.3	550.4±2.8
(c)	82.1±1.2	150.8±1.5	317.3±3.6	416.8±5.5	89.1±0.6	183.1±0.9	411.9±2.3	554.7±2.8



1.3
1.2
1.1
1.0
σ(exp.)/σ(calc.)
400.0
500.0
600.0
700.0
800.0
lab. momentum (MeV/c)
Si
C
Ca
Si
C
Ca


K$^+$ 715 MeV/c
C
^{6}Li (x1/10)
3.0
2.0
1.0
0.0
-1.0
-2.0
0.0
10.0
20.0
30.0
40.0
50.0
$\log_{10} d\sigma/d\Omega$ (mb/sr)
c.m. angle (deg.)